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A power filter for the detection of burst sources of gravitational radiation in interferometric detectors

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We present a filter for detecting gravitational wave signals from burst sources. This filter requires only minimal advance knowledge of the expected signal: i.e. the signal's frequency band and time duration. It consists of a threshold on the total power in the data stream in the specified signal band during the specified time. This filter is optimal (in the Neyman-Pearson sense) for signal searches where only this minimal information is available.

I. INTRODUCTION

Currently, the best understood and most highly developed technique for detecting gravitational waves with interferometric detectors is matched filtering. This technique is optimal if the waveform to be detected is known in advance. There are, however, potentially important sources of gravitational radiation that are not well enough modeled to obtain reliable waveforms. Included in this category are binary black hole mergers, which have been discussed in some detail by Flanagan and Hughes [1] (FH), and supernovae [2].

In order to detect poorly modeled sources, new techniques must be developed. Clearly, these techniques must perform well with incomplete prior knowledge of the expected signal. A number of techniques are under active investigation [3].

The purpose of this article is to discuss one such technique, the power filter [1,4]. This filter only requires prior knowledge of the duration and frequency band of the signal. It is therefore well suited to detecting black hole merger signals, for which FH have estimated these parameters. Furthermore, we shall show in the following sections that this filter is optimal in the sense that it gives the highest probability of correctly detecting a signal for a given false alarm probability. Our treatment here is cursory; a more comprehensive description of the filter is in preparation [4].

II. THE POWER STATISTIC

Consider the output $h(t)$ of the interferometric gravitational wave detector. It is sampled at a finite rate $1/\Delta t$ to produce a time series $h_j = h(j\Delta t)$, where j is an integer. A segment of N samples defines a vector $\mathbf{h} = (h_j, \ldots, h_{(j+N-1)})$. This vector can be written as

$$\mathbf{h} = \mathbf{n} + \mathbf{s} \tag{2.1}$$

where $\mathbf{n}$ is detector noise and $\mathbf{s}$ is a (possibly absent) signal.

The noise is assumed to be stochastic. The vectors $\mathbf{n}$ and $\mathbf{h}$ are therefore described statistically. Statistical fluctuations lead to two types of errors in detecting a signal: false alarms, in which signals are detected when none are present, and false dismissals, in which signals are not detected when present. An optimal filter is defined to be one which minimizes false dismissals for a given false alarm rate.

Neyman and Pearson have shown that an optimal filter is one for which the likelihood ratio Λ is compared to a threshold [5,6]. The likelihood ratio is defined to be

$$\Lambda[\mathbf{h}] \equiv \int \mathcal{D}[\mathbf{s}] \, \frac{p[\mathbf{h}|\mathbf{s}]}{p[\mathbf{h}|\mathbf{0}]}, \tag{2.2}$$

where $p[\mathbf{h}|\mathbf{s}]$ ($p[\mathbf{h}|\mathbf{0}]$) is the probability of obtaining $\mathbf{h}$ given that a signal $\mathbf{s}$ is present (absent) and $\mathcal{D}[\mathbf{s}]$ is a measure on the space of signals.

The quantities $p[\mathbf{h}|\mathbf{s}]$ and $p[\mathbf{h}|\mathbf{0}]$ depend on the statistical properties of the noise. For convenience, we assume in this article that the noise is stationary and Gaussian*. We can therefore write the probability distribution for the noise as

$$p[\mathbf{n}] = C \exp\left[-\frac{\mathbf{n}\cdot\mathbf{n}}{2}\right], \tag{2.3}$$

where C is a constant prefactor and $\mathbf{n}\cdot\mathbf{n}$ an inner product, both of which are determined by the autocorrellation matrix of the noise. When a signal is present (absent) we have $\mathbf{n} = \mathbf{h} - \mathbf{s}$ ($\mathbf{n} = \mathbf{h}$), and can easily use Eq. (2.3) to find the integrand in Eq. (2.2)

$$\frac{p[\mathbf{h} \mid \mathbf{s}]}{p[\mathbf{h} \mid \mathbf{0}]} = \exp\left[(\mathbf{s}\cdot\mathbf{h}) - \frac{1}{2}(\mathbf{s}\cdot\mathbf{s})\right]. \tag{2.4}$$

The measure $\mathcal{D}[\mathbf{s}]$ in Eq. (2.2) reflects our prior knowledge about the signal. For those signal parameters about which we have no prior knowledge, we choose a measure which reflects our ignorance.

* However, other types of noise can also be considered [4].

Consider now the case where we know the time window and frequency band in which a signal occurs. The measure $\mathcal{D}[\mathbf{s}]$ restricts the integral in Eq. (2.2) to the projection $\mathbf{h}_{||}$ of $\mathbf{h}$ into the space of vectors with the give window and frequency band. Introducing the notation $A^2 = \mathbf{s}\cdot\mathbf{s}$, $\mathcal{E} = \mathbf{h}_{||}\cdot\mathbf{h}_{||}$ and $\mathbf{s}\cdot\mathbf{h}_{||} = A\mathcal{E}^{1/2}\cos\theta$, we rewrite Eq. (2.2) as

$$\Lambda[\mathbf{h}] = \int \mathcal{D}[\theta, A]\ \exp\left[A\mathcal{E}^{1/2}\cos\theta - \frac{1}{2}A^2\right]. \tag{2.5}$$

Since we claim no prior knowledge of θ, a suitable measure over θ is uniform over all possible angles between $\mathbf{h}_{||}$ and $\mathbf{s}$ (i.e. over an $\mathcal{N}$ sphere, where $\mathcal{N}$ is the dimension of the space of vectors with the required duration and frequency band), which reflects our lack of knowledge.

While a suitable measure over the signal amplitude A can also be deduced, it is unnecessary here. Instead, one constructs a locally optimal statistic [6], $\Lambda_{\mathrm{loc}}[\mathbf{h}]$, which is appropriate in the limit of weak signals. To construct this statistic, one expands the likelihood ratio (2.5) in a Taylor series about $A = 0$. The statistic is simply the first non-vanishing coefficient (excluding the A^0 coefficient) in the expansion. Expanding (2.5) and integrating over θ we get

$$\Lambda_{\mathrm{loc}}[\mathbf{h}] \propto\ \mathcal{E} + \text{terms independent of } \mathbf{h}. \tag{2.6}$$

The terms independent of $\mathbf{h}$ clearly do not discriminate between the presence and absence of a signal. Thus, the optimal statistic for detecting a signal of known duration and frequency band is simply the total power in the detector output over that time and band.

III. OPERATING CHARACTERISTICS

In the previous section we determined the optimal statistic $\mathcal{E}$ for signals of known duration and frequency band in stationary Gaussian noise. We construct the optimal filter from this statistic via a threshold decision rule. That is, we calculate at what value $\mathcal{E}^\star$ of the statistic we incur the largest acceptable false alarm probability. We then compare values of $\mathcal{E}$ calculated from our data with $\mathcal{E}^\star$. A signal is said to have been detected if $\mathcal{E} > \mathcal{E}^\star$.

For Gaussian noise, false alarm and false dismissal probabilities can be calculated analytically up to quadrature. If no signal is present, $\mathcal{E}$ is just the sum of the squares of $V \equiv 2\times\delta t\times\delta f$ independent Gaussian random variables. Thus $\mathcal{E}$ has a χ^2 distribution with V degrees of freedom, and the false alarm probability for a value $\mathcal{E}^\star$ is just

$$P(\mathcal{E} > \mathcal{E}^\star \mid A = 0) = \frac{\Gamma(V/2, \mathcal{E}^\star/2)}{\Gamma(V/2)} \tag{3.1}$$

where $\Gamma(a, x) = \int_x^\infty e^{-t}t^{a-1}dt$ is the incomplete Gamma function.

If a signal of amplitude A is present, $\mathcal{E}$ is distributed as a weighted sum of χ^2 probability distributions,

$$p(\mathcal{E} \mid V, A) = \sum_{n=0}^{\infty} \frac{e^{-A^2/2}(A^2/2)^n}{n!}\frac{e^{-\mathcal{E}/2}(\mathcal{E}/2)^{n+V/2-1}}{\Gamma(n+V/2)}. \tag{3.2}$$

This is the non-central χ^2 probability distribution. The false dismissal probability is given by

$$P(\mathcal{E} < \mathcal{E}^\star \mid A) = \int_0^{\mathcal{E}^\star} p(\mathcal{E} \mid V, A)\ d\mathcal{E} \tag{3.3}$$

This probability can be integrated numerically.

IV. SUMMARY AND DISCUSSION

We have presented here a power filter to search for gravitational wave signals from burst sources in interferometric data. The filter is designed to look for signals of known duration and frequency bandwidth; when this is the only available information, and the noise is stationary and Gaussian, the power filter is optimal. Moreover, this filter is locally optimal for a wide class of non-Gaussian noise, thus making it a useful tool to analyze real interferometer data.

One shortcoming of the power filter is its inability to distinguish between gravitational wave signals, and spurious instrumental artifacts which produce time and band limited signals. This is mitigated by using multiple instruments to detect gravitational wave bursts. As an added benefit, bursts identified as noise can then be used for detector characterization. The extension of the power filter to multiple instruments will appear in an upcoming article [4]; this article also contains a more complete discussion of the filter, including implementation strategies and a comparison to matched filtering.

In conclusion, we think that the power filter provides a useful tool for gravitational wave data analysis. It should play a significant role in detector characterization for single interferometers, and should form the basic building block in an hierarchical detection strategy using multiple interferometers.

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